ILL-POSEDNESS OF THE SAINT-VENANT-HIRANO MODEL

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Key words River morphodynamics, mixed size sediment, ellipticity.

The morphodynamic evolution of a river is generally modelled using a set of conservation equations for water mass and momentum and sediment mass. For unisize sediment this set of conservation equations comprises the Shallow Water Equations and the Exner equation. For modelling processes associated with mixed-size sediment (e.g. armoring, downstream fining) it is necessary to introduce a conservation equation for each grain size fraction. The first and most widely applied model is that of Hirano [1], i.e. the active layer model. A necessary condition for the model to be representative of physical processes is that the set of partial differential equations is well-posed. In the case of a time-evolving problem the system of equations must be of the hyperbolic (or parabolic) type, which is ensured by real eigenvalues, i.e. the information must propagate at a real celerity. Yet, using the Hirano model the system's character may become elliptic [2], which happens when an eigenvalue becomes complex. We have conducted an analysis of the system of equations to gain insight into its well-posedness. To that end we assume quasi-steady one-dimensional flow over a bi-modal sediment mixture. We then obtain analytical expressions of the eigenvalues (λ) which can be written as a combination of two celerities: the De Vries celerity (of disturbances that arise due to spatial gradients in the bed load transport rate that are unrelated to mixed sediment) and the Ribberink celerity (of those disturbances that originate from the presence of mixed sediment). When these two celerities are similar (Figure 1a), the discriminant of the eigenvalues (Δ) may become negative (Figure 1b) and the system loses its hyperbolic character. To study the consequences of the elliptic behavior we conduct a perturbation analysis and we run numerical simulations. A solution of a river degrading into a fine substrate (Simulation 1, Figure 1c) is elliptic for certain values of active layer thickness (Simulation 2, Figure 1d), which result in oscillations that are not related to physical mechanisms. If mixed-size sediment simulations suffer from ellipticity the results are not only unreallistic but also largely affected by the domain discretization. Paradoxically, a more detailed discretization of the domain, which leads to more accurate results, increases the velocity at which oscillations grow. Also, the absence or presence of oscillations is no guarantee of elliptic or hyperbolic character and the representativeness of the results can only be assured by a specified mathematical assessment of the system's character.

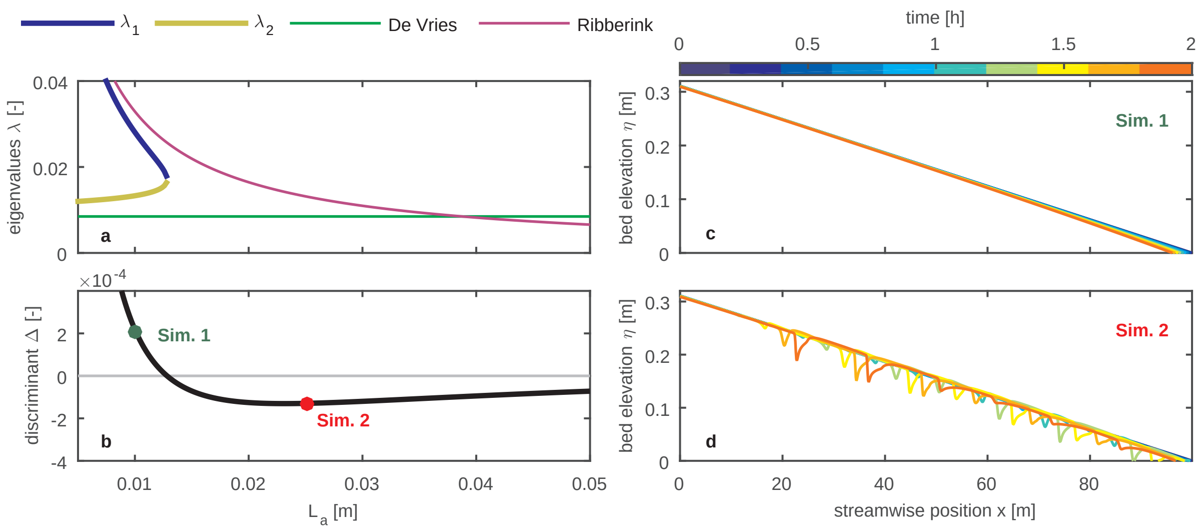

Figure 1: Eigenvalues (**a**), and discriminant (**b**) for a varying active layer thickness. Numerical results of a hyperbolic run (Simulation 1, **c**) and elliptic run (Simulation 2, **d**).

References

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