
UNCONVENTIONAL LIKELIHOOD FUNCTIONS IN GEOPHYSICAL INFERENCE

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Likelihood functions are at the basis of all modern inference. They play a pivotal role in statistical inference, are a key part of popular Bayesian Inference methods and when recast as data misfit functions form the central component of nonlinear optimization methods of inversion. In both Maximum Likelihood and Bayesian Inference approaches the definition of the Likelihood has a major influence on the solution of an inverse problem and yet in many studies its form receives relatively little attention. The role of the Likelihood function is to measure the *probability of the data given the model*, which requires the statistics of the noise to be known. However in many cases data noise characteristics are not well known, and guesses have to be made. A particularly common Likelihood crime is to choose its form for convenience e.g. by ignoring correlations in the noise between successive data measurements in a time series, or by assuming quantities that are measured, and hence have associated noise, are error free auxiliary variables. A common situation in regression problems within the geosciences. Each of these crimes are committed to simplify the Likelihood often in the hope that the solution with Maximum Likelihood or Maximum *a posteriori* PDF is not too affected by the approximations introduced. Even when this is true, however, estimates of uncertainty and also goodness of fit can be drastically influenced by such approximations.

In this paper we look at two unconventional approaches to building Likelihood functions. In the first we derive and justify a Likelihood function for time series fitting problems with arbitrary data errors, i.e. where both parameters y and t (or x) are measured and contain error which may be correlated between components of a single datum or correlated between data, and may be multi-dimensional Gaussian or otherwise. We illustrate use of the Likelihood function in a 1-D changepoint problem. [?]. A comparison is made between results of Maximum Likelihood and Bayesian Partition modelling. We also apply the latter to reconstruction of relative sea-level over the past five glacial cycles, 500, 000 years before present based on published $\delta^{18}O$ measurements [?]. We show that using the unconventional Likelihood here provides robust uncertainty measurements on both relative sea level height and its gradient.

The second unconventional Likelihood involves utilizing concepts from Compressive Sensing. Here we build Likelihoods from random sub-samples in time of a seismic waveform time series and examine the constraint that may be placed on an Earth model. The results suggest that large portions of a seismic waveform may be discarded without significant loss in the constraint provided by the Likelihood. This may suggest new ways to approach full seismic waveform inversion.

References

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