GRADUAL WAVELET RECONSTRUCTION (GWR) FOR UNDERSTANDING THE NONLINEAR STRUCTURE OF TIME-SERIES DATA AND PLACING CONFIDENCE ON MODEL PARAMETERS

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Gradual wavelet reconstruction (GWR) is a data simulation method and this presentation will cover its formulation as well as geophysical applications. Surrogate data are extremely useful for testing hypotheses about data that are anticipated to be nonlinear, e.g. "is my time series chaotic?". Given an appropriate metric (e.g. maximal Lyapunov exponent), the value of the metric for the data is compared to the synthetic, linear, surrogate time series and a significant difference at the level α exists using a two-tailed test if the value for the original data is smaller or larger than that for $\left(\frac{2}{\alpha}\right) - 1$ surrogate series.
Fourier phase randomisation is the classic way to generate such linear surrogates. However, upon rejection of the null hypothesis (the data are nonlinear by the chosen metric) one cannot readily investigate how nonlinear a dataset is, or set up computational models with the appropriate degree of nonlinearity. GWR defines a continuum $0 \leq \rho \leq 1$ where the lower limit is the phase-randomised case, and the upper is the original data [1]. Using a wavelet transform to systematically preserve aspects of the phase in the original data, surrogates can be generated along this continuum. This permits a suite of new research possibilities:

- Generate synthetic river discharge data with the correct degree of asymmetry to the hydrograph (steep rising limb and gradual falling limb) to provide a new way to place confidence limits on extreme floods [2];
- Generate inlet boundary conditions for numerical simulations, and examining the sensitivity of the simulation to this perturbation [6];
- Show that the key nonlinearity driving differences in sediment entrainment by coherent flow structures is the frequency of occurrence of the structure, not the stress distribution [3];
- Show the the shape of gravel bed-forms in a channel increase in complexity with discharge and that the results are frame-of-reference dependent [4];
- Demonstrate that model parameters with small absolute values (in our case, a Fokker-Planck equation for the evolution of the velocity increments in turbulence) are significantly different to zero and are therefore dynamically relevant [5].

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References

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