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## WHAT DOES THE HESSIAN OPERATOR TELL US ABOUT UNCERTAINTIES AND OPTIMAL EXPERIMENTAL DESIGN?

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We discuss and apply methods for uncertainty quantification and optimal experimental design in time-domain full-waveform inversion based on a low-rank approximation of the Hessian operator. The fast decaying spectrum of the Hessian operator, which is a well-known property for problems in full-waveform inversion, motivates a parameterization using a small number of eigenvectors corresponding to the dominant eigenvalues of the Hessian. This only requires a few Hessian-vector products, which can be efficiently computed using adjoint methods at the cost of two additional simulations per vector. The resulting low-rank approximation of the Hessian has plenty of useful applications in the context of joint parameter inversion in elastic medium with multiple seismic sources. In particular, we focus on (i) steering the optimal experimental design of seismic events, (ii) providing information on uncertainties, and (iii) preconditioning the Newton system to speed up the solution of the deterministic inverse problem.

The separable structure of the multi-experiment inverse problem allows us to detect redundancies in the data and to gradually include more events. Furthermore, these techniques can be used to assess the quality of source-encoding strategies. In particular, we can extract information from the low-rank approximation of the Hessian to steer the selection of weights during the stacking of individual point sources.

In the context of uncertainty quantification, we utilize the low-rank approximation to construct an estimate of the covariance matrix. From this, we extract information on (i) spatial variance reduction, (ii) multi-parameter tradeoffs and (iii) spatial correlations. Furthermore, since the number of unknowns is reduced by several orders of magnitude compared to the usual nodal discretization, this method enables the use of sampling-based strategies like Markov Chain Monte Carlo methods.

We demonstrate the applicability of the approach as well as its limitations in numerical examples for 3D full-waveform inversion.

### References

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