HIERARCHICAL BRANCHING PROCESSES

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Nature produces many branching tree-like structures beyond the botanical trees. Despite their apparent diversity, a large number of rigorously studied natural branching structures exhibit simple two-parametric Tokunaga self-similarity and Horton scaling. The Horton scaling is a weaker property that addresses the principal branching in a tree; it is a counterpart of the power-law size distribution for system's elements. The stronger Tokunaga self-similarity addresses so-called side-branching; it ensures that different levels of a hierarchy have the same probabilistic structure (in a sense that can be rigorously defined). The solid empirical evidence motivates the search for a flexible and conveniently treatable class of models that exhibit Horton and Tokunaga self-similarity.

We introduce a class of stochastic processes that we call hierarchical branching processes. By construction, the processes satisfy the Tokunaga, and hence Horton, self-similarity constraints. Taking the limit of averaged stochastic dynamics, we obtain the deterministic system of differential equations that describe the temporal dynamics of a Tokunaga branching system. In particular, we study the averaged tree width function to establish a phase transition in the Tokunaga dynamics that separates fading and explosive branching. We then describe a class of critical Tokunaga processes (that happen at the phase transition boundary) that includes as a special case the celebrated critical Galton-Watson branching process. We illustrate efficiency of the critical Tukunaga processes in describing diverse observed dendritic structures, and discuss the related critical phenomena from the point of view of respective applications.