DESTABILISATION OF SHEAR FLOWS
BY ALFVÉN WAVES AT LOCALISED MAGNETIC FIELDS

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The instability of shear flows in the presence of magnetic fields is fundamental to understanding a wide range of geophysical and astrophysical phenomena. We investigate the simplest paradigm problem of interest, which is the instability of plane parallel shear flows with aligned field, to two-dimensional disturbances. It is well known that magnetic fields are stabilising in the sense that a sufficiently strong and widespread field always implies stability, even if the underlying shear flow is hydrodynamically unstable [1, 2]. We focus on cases where the shear flow has no inflexion points and is thus hydrodynamically stable, and show how such flows can nevertheless be destabilised by the addition of one or more thin regions of magnetic field.

The simplest such instability occurs when the basic flow has uniform shear (constant vorticity), and two thin regions of magnetic field are added. An explicit analytical solution for the linear instability is presented when the magnetic fields are idealised as having infinitesimal width, showing that there is always instability for some range of along-stream wavenumbers. The strength of the instability is reduced for the more realistic case of magnetic fields of finite width; the linear stage is investigated analytically using matched asymptotic expansions, and the nonlinear stage is investigated numerically using standard pseudospectral techniques. The instability can be unambiguously attributed to the mutual amplification of a pair of counter-propagating Alfvén waves, with one wave propagating along each thin region of magnetic field.

A distinct instability occurs when the basic flow has two regions of uniform shear separated by a thin vorticity gradient, and one thin region of magnetic field is added. An explicit analytical solution can be obtained for the linear instability when the vorticity gradient and magnetic field are both idealised as having infinitesimal width, which can be extended numerically to smooth profiles. This instability can be attributed to the mutual amplification of a Rossby wave (at the vorticity gradient) and an Alfvén wave (at the magnetic field).

Both of these instabilities are interesting extensions to magnetohydrodynamics of well-known shear instabilities in atmosphere-ocean fluid dynamics, involving interactions between various combinations of Rossby waves and internal gravity waves (e.g., [3, 4, 5]). Like their hydrodynamic counterparts, these magnetohydrodynamic instabilities can also be interpreted in terms of positive and negative energy waves (cf. [6]). For planetary flows, they could act as a primary instability of appropriately structured toroidal fields, or as a secondary instability on thin filaments of magnetic field drawn out by other processes.

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