

OPTIMAL TRANSPORT DISTANCE FOR SEISMIC TOMOGRAPHY: APPLICATION TO FULL WAVEFORM INVERSION

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Full waveform inversion (FWI) is a high resolution seismic imaging technique based on the minimization of the L^2 distance between observed and predicted seismic data. An initial estimation of the subsurface model is iteratively improved following local optimization techniques. When applied to the reconstruction of subsurface velocity parameters, the method is known to suffer from cycle skipping. Velocity models matching the observed data up to one or several phase shifts correspond to local minima of the L^2 misfit function. Avoiding to converge to these local minima requires a sufficiently accurate initial velocity model, and adequate data pre-processing: hierarchical approaches privileging the lowest components of the data and the transmitted energy are usually implemented for real data applications [Virieux and Operto, 2009].

In a recent study, we have propose to use an optimal transport distance for measuring the misfit between observed and predicted data [Métivier et al., 2016]. Optimal transport originates from the work of the French engineer Gaspard Monge (1790) in an attempt to devise the best strategy to move sand to a building site, in a prescribed configuration. In more modern mathematics language, the optimal transport distance between two mass distributions $\mu(x)$ and $\nu(y)$ is the solution of a minimization problem among the set of the mapping transporting $\mu(x)$ over $\nu(y)$, given a cost $c(x, y)$ measuring the effort for moving a unit mass from x to y [Santambrogio, 2015]. Such a distance has been successfully used in image processing for color histogram matching, pattern recognition, and shape identification [Lellmann et al., 2014]. The ability of this distance to detect shifted patterns in two distinct images is an interesting property for FWI. A better convexity of the associated misfit function with respect to time-shifts is expected, which should efficiently mitigate cycle skipping issues.

In its conventional formulation, optimal transport distance is dedicated to the comparison of positive signals with a strict mass conservation. Theses assumptions (positivity, mass conservation) are not satisfied by predicted and observed seismic data. For this reason, the use of a modified optimal transport distance, based on the Kantorovich-Rubinstein norm, is investigated. An efficient numerical strategy based on a proximal splitting method is implemented [Combettes and Pesquet, 2011], which offers the possibility to address large-scale problems associated with realistic 2D and 3D FWI applications. An example of application on the 2D Marmousi case study is provided in Figure 1. Starting from a crude initial model, conventional L^2 FWI fails in recovering the subsurface velocity. The strategy based on the optimal transport distance provides a significantly better subsurface velocity estimation. Future work will be dedicated to the application of this strategy on the 2D Nankai and the 3D Valhall data-sets.

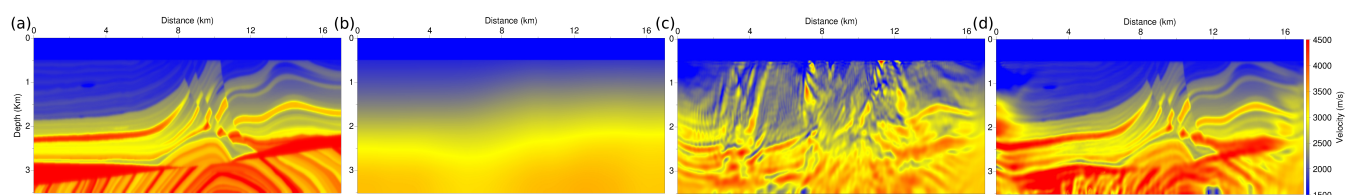


Figure 1. Marmousi model (a). Rough Initial model (b). FWI estimation using the L^2 distance (c). FWI estimation using the optimal transport distance (d).

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