SOLVING SPATIAL INVERSION PROBLEMS USING EXACT SAMPLING

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Key words Bayesian, recursive, geology, geophysics, inversion, sampling, Monte Carlo, Markov chain, posterior, prior

Geoscientists often use spatially discretized cellular models of the Earth where data in each grid cell provide independent information about the model parameters of interest at that location, yet where parameters are spatially correlated. In Bayesian inference the data-derived information is given as a set of likelihoods describing the (unnormalized) probability of the parameters, given only the data in each cell. Pre-existing information about the model parameters' values and their spatial correlations may be described by a prior probability distribution. The prior, likelihoods, and Bayes' rule together specify a posterior probability distribution that describes the resultant state of information over all model parameters. However, due to the high dimensionality of typical models, the posterior is usually only known up to a multiplicative constant and only at specific, numerically evaluated points or samples in the model space (i.e., it is not known analytically). Markov chain Monte Carlo (McMC) methods are typically used to produce an ensemble of correlated samples which converges to the posterior as the number of samples tends to infinity. These ensembles are often slow to converge in distribution to the posterior; indeed, they may not converge in finite time, and detecting their state of convergence is often impossible in practice. Thus, estimates of the posterior obtained from such samples may be substantially biased.

We derive a recursive algorithm which samples the posterior exactly, providing independent (uncorrelated) posterior samples from the first sample onwards [1]. Thus the method avoids the above convergence issues completely. Its computational cost scales with the size of the parameters' sample space, the prior's spatial range of dependency, and the shortest edge dimension of the grid. We develop an approximation to the algorithm such that it may be used on large 2-D (and potentially 3-D) model grids.

We apply the algorithm to invert synthetic seismic attribute data (impedances) for subsurface structure, by combining the attribute data with Geological prior information that includes spatial correlations expected of real geological strata. We obtain results which compare favourably to the results of McMC (Gibbs) sampling – indeed the latter exhibits convergence problems and bias.

This method might be generalised to other types of problems as will be discussed, providing an alternative to McMC methods for a wider class of problems.

References

[1] M. Walker & A. Curtis, Spatial Bayesian inversion with localized likelihoods: An exact sampling alternative to McMC, J. Geophys. Res.. **119**, 5741-5761 (2014). Doi: 10.1002/2014JB011010